Combining Functions

Chapter 2.6 covers this information on page 272 – 279: Combinations of Functions in Blitzer, *College Algebra Essentials* 3e. We are simply adding two equations together to get a third equation and then defining all the numbers that could be used to get an answer. We are working with the concept of “functions” where the answers relate to each other on a graph; we have an $x$ and a $y$ as a coordinate. In these cases we are defining real numbers (no imaginary numbers). Examples of the graphs formed from common algebra equations or functions are shown on page 255.

I’m going to walk through the problem found on page 273 which is similar to the one you have for your EYB. Let $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$. Now, let’s solve the following problems:

a. $f + g$: If you look on pg 272, you will see that to find the sum of $f$ and $g$ plug the numbers into the function. You may see $f + g$ written as $(f + g)(x)$. Either way, $f + g$ can be re-stated as:

$$ (f + g)(x) = (2x-1) + (x^2 + x - 2) $$

Now let’s solve the equation by combining like terms:

$$ (f + g)(x) = (2x-1) + (x^2 + x - 2) $$

First, I’ll write this one out in order of operations:

$$ x^2 + 2x + 1x -1 - 2 $$

So when we combine the like-terms, we get:

$$ (f + g)(x) = x^2 + 3x - 3 $$ (This is the definition of $f + g$).

Now that we have our definition or our new equation, we have to tell everyone what numbers they can use in the equation that work to give us a function. This part is quite simple, here’s some rules to follow to define the number that can be used, or the *domain*.

1. When combining two functions to get a new equation or the definition, if the equations do not involve division, then the answer is the set of all real numbers, written $(-\infty, \infty)$. This also says negative infinity to infinity. This is the answer to the domain of $(f + g)$.

2. If the combination of the two functions results in division, (we will see this soon), the answer can’t give us a “0” as a denominator because that is not a real number; so we will have to figure out what numbers make the denominator 0 and exclude those from the answer. As a quick example, let’s say our final definition is $1/x - 1$. We have to exclude 1 from our answer because $1-1 = 0$, and $1/0$ is not a real number. So our answer would be all real numbers, excluding 1, written $(-\infty,1) \cup (1,\infty)$. This says negative infinity to 1 union 1 to infinity, or all numbers to 1 then all numbers above 1 (but not 1!).

I’m going to skip the $f - g$ because it is solved the same way we did the $f + g$ (except, of course you subtract the two equations instead of adding them. I’m also skipping the $f \cdot g$ (f times g), because it works the same way too.
So let’s look at

d. $\frac{f}{g}$: If you look on pg 272, you will see that to divide $f$ and $g$ plug the numbers into the function.

You may see $\frac{f}{g}$ written as $(\frac{f}{g}) (x)$. We will re-state this just like we did in the addition, by putting the two equations together:

$$(\frac{f}{g}) (x) = \frac{(2x-1)}{(x^2 + x - 2)}$$

This function cannot be simplified any further, so this is our definition.

Now for the answer to the domain of $\frac{f}{g}$: remember that we cannot have a “0” as the denominator, so let’s solve the denominator for “0” and see what numbers to exclude from our domain.

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0$$

$$x + 2 = 0 \quad x - 1 = 0$$

$$x = -2 \quad x = 1$$

These are the two numbers we have to exclude from our answer, or the domain of our problem. So this will be written $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

This says: negative infinity to negative 2 union negative two to 1 union one to infinity.

This means: all numbers to -2, and all numbers between – 2 and 1 and all numbers from 1 to infinity (but not -2 or 1)!