Factoring a Polynomial

Problem: \(14x^2 + 3x = 2\)

We know this is a 2\(^{nd}\) degree polynomial equation because of the \(x^2\).

We know this is a quadratic equation because we can write this equation in a standard form of \(ax^2 - bx - c\) if we set the equation to equal 0.

We have some choices to make here, we can solve a quadratic equation in three different ways: factoring, using the square root property or using the quadratic formula.

We can eliminate the square root property because we cannot use that unless there is a 0x (or no x factor in the problem – we have a 3x).

Solve the quadratic equation using factoring: \(14x^2 + 3x = 2\)

First we will set the problem = 0 by subtracting 2 from both sides, placing it in the form of a quadratic equation \(ax^2 + bx + c = 0\):

\[
14x^2 + 3x - 2 = 0
\]

\[
14 = a \quad 3 = b \quad -2 = c
\]

Now we will find two factors for the x term to use grouping. We need to find 2 numbers whose product is a \(x\) \(c\) or in this case:

\[(14) (-2) = -28\]

The best way to do this is to build a chart for showing the factors of 28.

<table>
<thead>
<tr>
<th>FACTORS of 28</th>
<th>If I add them I get</th>
<th>If I subtract them I get</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 28</td>
<td>29</td>
<td>27</td>
</tr>
<tr>
<td>2 14</td>
<td>16</td>
<td>2</td>
</tr>
<tr>
<td>4 7</td>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>
From the chart, you can see that if you use 4 and 7, you can factor the equation and get \(3–\) which happens to be the “\(b\)” in the equation.

So now we can substitute the factors \(+7x\) and \(-4x\) in place of the \(3x\) (because \(7x-4x = 3x\)) and we can get two factorable equations:

\[
14x^2 + 7x - 4x - 2 = 0
\]

**Now we group:** I put the 1 on the outside of the ( ) in the second equation:

\[
(14x^2 + 7x) - 1(4x + 2) = 0
\]

**WATCH YOUR SYMBOLS:** Notice, for the second equation, if the outside is a – symbol, we have to change the sign on the inside (distribute the negative). The easiest way to see this is by putting a -1 on the outside of the ( ) and making sure that when you multiply the -1 through, your symbols are the same as we had for the beginning equation.

**Factor each small group**

\[
(14x^2 + 7x) = 7x(2x+1)
\]

and

\[-1(4x + 2) = -2(2x+1)\]

So

\[7x(2x+1) - 2(2x+1) = 0\]

**Now we can factor again.** The problem above is two binomials. We have a common factor for both \((2x+1)\), so we can re-write the factorization in this manner:

\[(7x - 2)(2x + 1) = 0\]
We are almost there! **We need to set each equation to 0 and solve**

\[
\begin{align*}
7x - 2 &= 0 \\
7x &= 2 \\
7x &= 2
\end{align*}
\]

\[
\begin{align*}
2x + 1 &= 0 \\
2x &= -1
\end{align*}
\]

\[
\begin{align*}
x &= 2/7 \\
x &= -\frac{1}{2}
\end{align*}
\]

**AND THESE ARE THE ANSWERS!**

Always check your work!

\[
\begin{align*}
14(\frac{2}{7})^2 + 3(\frac{2}{7}) &= 2 \\
14(\frac{4}{49}) + 3(\frac{2}{7}) &= 2 \\
56/49 + 6/7 &= 2 \\
56/49 + 42/49 &= 2 \\
98/49 &= 2 \\
1 &= 2
\end{align*}
\]

**AND**

\[
\begin{align*}
14(-\frac{1}{2})^2 + 3(-\frac{1}{2}) &= 2 \\
14(\frac{1}{4}) + 3(-\frac{1}{2}) &= 2 \\
14/4 - 3/2 &= 2 \\
14/4 - 6/4 &= 2 \\
8/4 &= 2 \\
2 &= 2
\end{align*}
\]