Type I and Type II Errors
Objectives

• Review the use of statistics to estimate population parameters
• Review hypothesis testing to determine the significance of statistics
• Discuss the potential for chance and chance error in estimation
• Identify the 2 types of error: Type I and Type II
• Associate the types of error with the calculation of power and sample size
Parameter vs. Statistic

**Parameter**
- Based on the population
- An absolute number
- Example: Data in the form of birth certificates is collected on every person born in the U.S. Therefore, Natality Records are population level data and all information computed using that data are parameters.

**Statistic**
- Based on a sample of the population
- An *estimate* of the parameter it represents
- Example: Data in National Surveys such as NHANES or BRFSS is collected from a random sampling of the U.S. population. All information computed using that data are statistics.
Hypothesis Testing

• You have seen how to use statistics to estimate the 95% confidence interval of a variable and also how to interpret this estimate to learn something about the population the sample was drawn from.

• You have also seen that testing a hypothesis is another way to use sample data to learn about your population.

• In both cases these cases, statistic is an estimate and so has a possibility of error built into it.

• The null hypothesis is frequently represented by $H_0$ and is the hypothesis you are testing.

• The alternative hypothesis is $H_A$ or $H_1$.

• You test a null hypothesis of this sort by drawing a random sample from the population in question and calculating an appropriate statistic on the variables of interest.

• If you obtain a value of the statistic that would occur rarely through chance alone if the null hypothesis is really true, you would have reason to reject that hypothesis.
The Null Hypothesis

• The null hypothesis is often quite different from the research hypothesis
• It is usually the theory that no association exists between the independent and dependent variables
• In the null hypothesis you make an assumption that the results observed are no different from those expected on the basis of chance alone
• You write the null hypothesis so that it is testable – you can reject or fail to reject the null hypothesis based on statistical significance
Null vs. Alternative Hypothesis

Example:

- Null Hypothesis
  \[ H_0: \text{Mean blood pressure on drug A} = \text{Mean blood pressure on drug B, as measured 6 hours after start of treatment} \]

- Alternative Hypothesis
  \[ H_1: \text{Mean blood pressure on drug A} < \text{Mean blood pressure on drug B, as measured 6 hours after start of treatment} \]

It would be a better hypothesis if you can specify how much lower mean blood pressure should be to be clinically significant, example: 5 mmHg
Failure to Reject

• When testing the null hypothesis we first assume it is true – so we either reject or fail to reject it based on the analysis.
• This is based on the philosophical idea that you cannot prove a negative – you can only disprove or fail to disprove it.
• We compare the outcome we observed to the outcome that could be expected by chance alone. Sampling only allows us to estimate parameters and this means there is a possibility of error.
• If the probability of obtaining, by chance, an association of two or more variables as strong as the one observed, is very small, we can reject the null hypothesis. If that probability is large, we fail to reject the null hypothesis.
Error and Chance Error

• Error is a false or mistaken result obtained in a study or experiment.

• There are two main types of error:
  – Random error: just as it says this error is random and not connected in any way to the methodology of the study or the way the population was sampled. This error is generally regarded as due to CHANCE.
  – Systematic error: this type of error often has a recognizable source, either related to the method or to the sampling. This is also often called a bias. An example is selection bias which is a systematic error that results because of how you sampled your population.

• Chance error can be further described as either Type I or Type II, based on what effect the error has on your interpretation of the statistical results.
Types of Chance Error

Type I
• The chance or probability that you will reject a null hypothesis that should not have been rejected.
• This will result in you deciding two groups are different or two variables are related when they really are not.
• The probability of a Type I error is called $\alpha$.

Type II
• The chance or probability that you will not reject a null hypothesis when it should have been rejected.
• This will result in you deciding two groups are not different or two variables are not related when they really are.
• The probability of a Type II error is called $\beta$. 
### Examples

<table>
<thead>
<tr>
<th>Null Hypothesis is True</th>
<th>Null Hypothesis is False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject the Null Hypothesis</td>
<td>Type I Error ($\alpha$)</td>
</tr>
<tr>
<td>Do Not Reject the Null Hypothesis</td>
<td>Correct Decision</td>
</tr>
</tbody>
</table>

You sample two populations and test the null hypothesis that population A has the same rate of bladder cancer than population B. You compare your samples and reject the null hypothesis and state that the rates are different. In reality the rates are the same but you made a Type I error due to the way you sampled the two populations.

You sample two populations and test the null hypothesis that population A has the same rate of bladder cancer than population B. You compare your samples and fail to reject the null hypothesis and state that the rates are not different. In reality the rates are different but you made a Type II error because your samples were not large enough.
Type II Error and Power

• In the previous slide’s second example you learned that you can have a Type II error if your sample size is not large enough.

• Power, which is the ability to detect differences if they exist, is the inverse function of Type II error or $\beta$.

• Power = $1 - \beta$. A typical minimum for power is 80%, so the largest preset for $\beta$ is 0.20. As with $\alpha$, smaller is better as it allows less opportunity for chance error.

• In the next slide you will see how your preset values for $\alpha$ and $\beta$ affect your required sample size.

• Sample size is very important because with an underpowered study you could fail to reject a null hypothesis that should be rejected.
Type I and II Error and Sample Size

For a Continuous (Normal) Distribution:

\[ 2n = \frac{4(Z_\alpha + Z_\beta)^2 \sigma^2}{\delta^2} \]

- Where \( Z_\alpha \) is the value of \( Z \) at the preset \( \alpha \)
- \( Z_\beta \) is the value of \( Z \) at the preset \( \beta \)
- \( \sigma^2 \) is the pooled variance of the samples
- \( \delta \) is the clinically significant effect size you wish to observe (difference between the means)

You do not need to know the above formula or use it during this class to calculate your sample size. What you do need to know is that both the preset \( \alpha \) and the preset \( \beta \) are directly related to the sample size. As you set those lower, the \( Z \) value associated with them gets higher (look in the book at the \( Z \) tables if you doubt this) and therefore the sample size required gets larger as well. The more you want to protect against the possibility of error, the larger your sample will need to be.